Data Locality Optimizations to Improve the Efficiency of Multigrid Methods

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The Paradox

Solving scalar elliptic PDE with FMV

Theoretically:

- 100 operations per unknown
- 10^9 operations per second on current RISC CPUs

  => 1024 x 1024 problem in less than 1 sec ?

mgd9v

Properties

- robust multigrid method
- sawtooth multigrid cycling
- optimized for vector computers
- Fortran 77 code

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Experiments

- solving Poisson's equation
- L_2-norm of residual < $4*10^{-8}$
- grid size 513 x 513 and 1025 x 1025
- on SGI PowerCh. with MIPS R8000 (75 MHz, Peak 300 MFlop/s)
- compiled with MIPS Fortran using "-fast=ip26 flag"

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Results

size	time	#Flop	MFlop/s	% Peak
513	19.7 sec	$271*10^6$	13.8	4.6
1025	76.3 sec	$1083*10^6$	14.3	4.7

=> 2 orders of magnitude slower than expected !

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SGI Origin 2000 Node: MIPS R10000 (195 MHz, 390 MFlops/s)

size	time	MFlop/s	% Peak
513	6.2 sec	43.7	11.2
1025	34.3 sec	43.7	11.2

DEC PWS 500 A21164 (500MHz, 1GFlop/s)

size	time	MFlop/s	% Peak
513	7.2 sec	36.6	3.6

Computer Technology

RISC CPUs

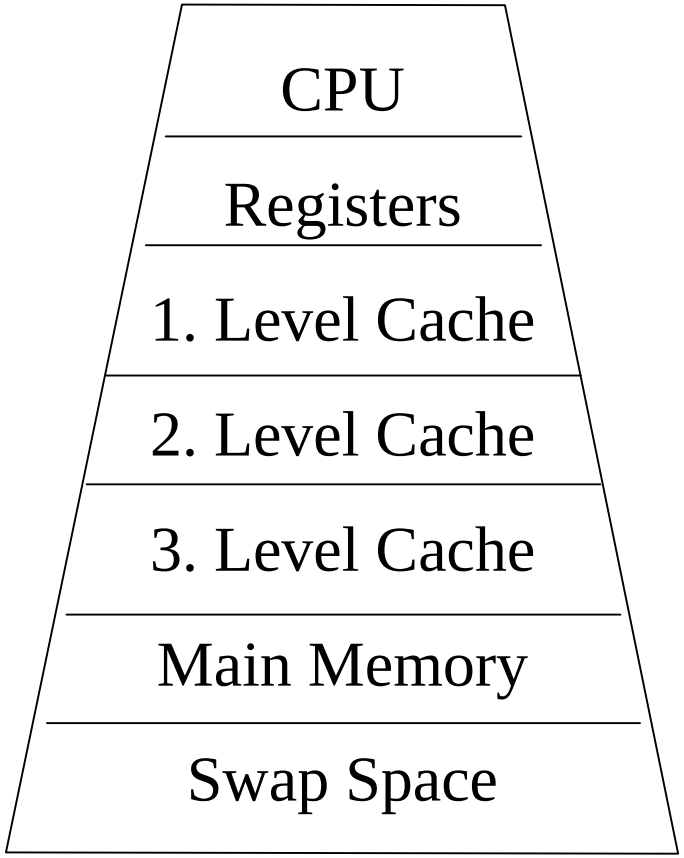
- pipelined execution with 5-10 stages
- execution of up to 4 floating point operations per cycle
- clock rates up to 600 MHz

Main Memory

- bandwidth up to 0.2 floating point operands per cycle
- latency higher than 60 cycles

Memory Hierarchy

Example: DEC PWS 600 A21164



	Capacity	Bandwidth	Latency
←	256 Byte	28800 MB/s	1.7 ns
←	8 KByte	19200 MB/s	1.7 ns
←	96 KByte	9600 MB/s	5.0 ns
←	2 MByte	870 MB/s	23.3 ns
←	1536 MByte	1070 MB/s	105.0 ns

Cache Aware Multigrid

- starting with straightforward multigrid code in C
  - constant finite difference stencil on each grid level
  - 9 point stencil
  - V-Cycle
  - FMV
  - red/black Gauss Seidel smoother
- in cache optimizations
  - loop unrolling
- data locality optimizations (in smoother and overall algorithm)
  - fusion
  - melting

Red-Black Relaxation

Standard

red nodes:

```
do i =1, n-1
    do j = 2-i%2, n-1, 2
        Relax( u(i, j) )
    endo
endo
```

black nodes:

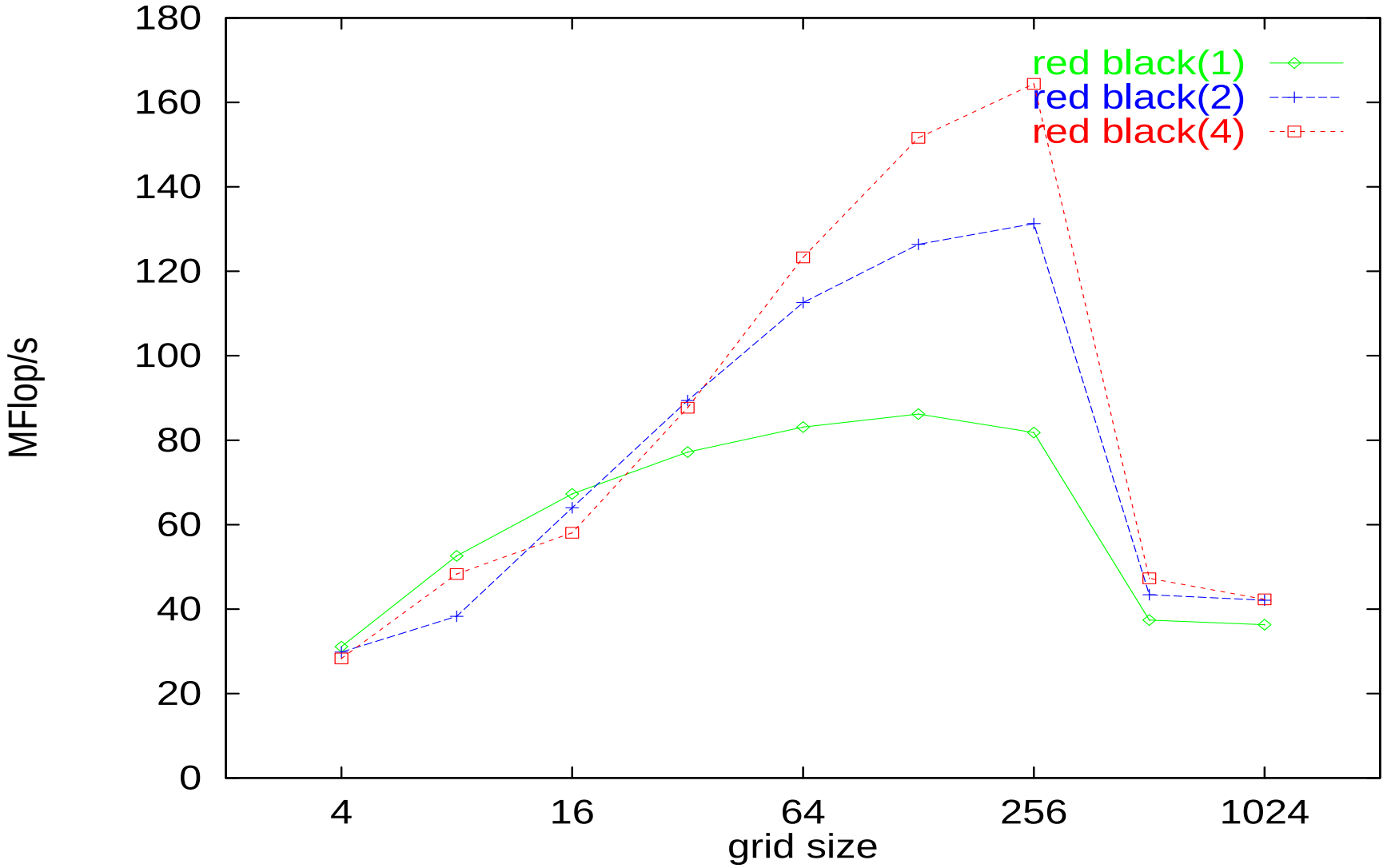
```
do i =1, n-1
    do j = 1+i%2, n-1, 2
        Relax( u(i, j) )
    endo
endo
```

Loop Unrolling (simplified)

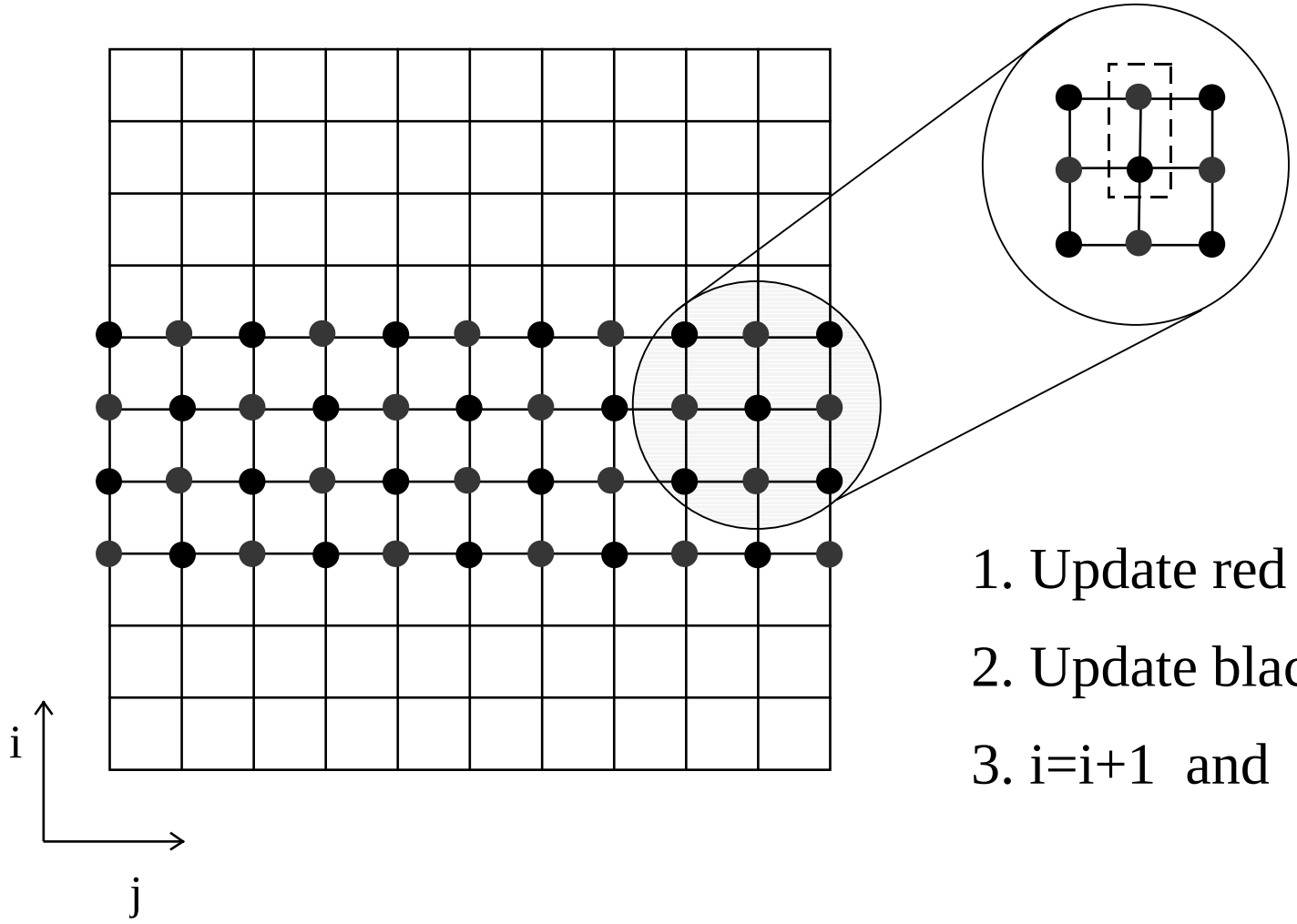
red nodes:

```
do i =1, n-1
    do j = 2-i%2, n-1, 4
        Relax( u(i, j) )
        Relax( u(i, j+2) )
    endo
endo
```

R/B Results

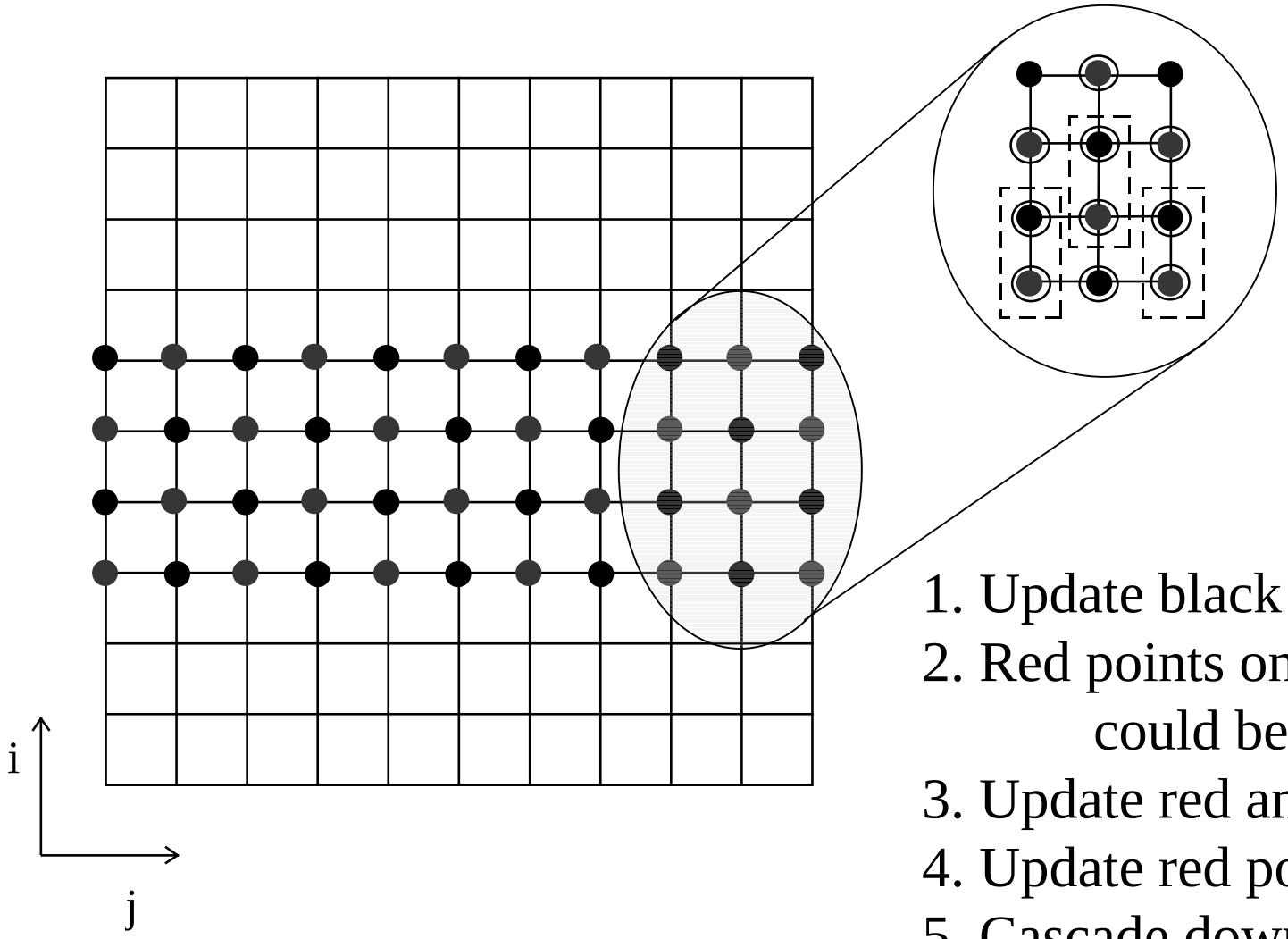

MFlop/s
180
160
140
120
100
80
60
40
20
0
4
16
64
256
1024
grid size
red black(1)
red black(2)
red black(4)

Cache Aware Relaxation



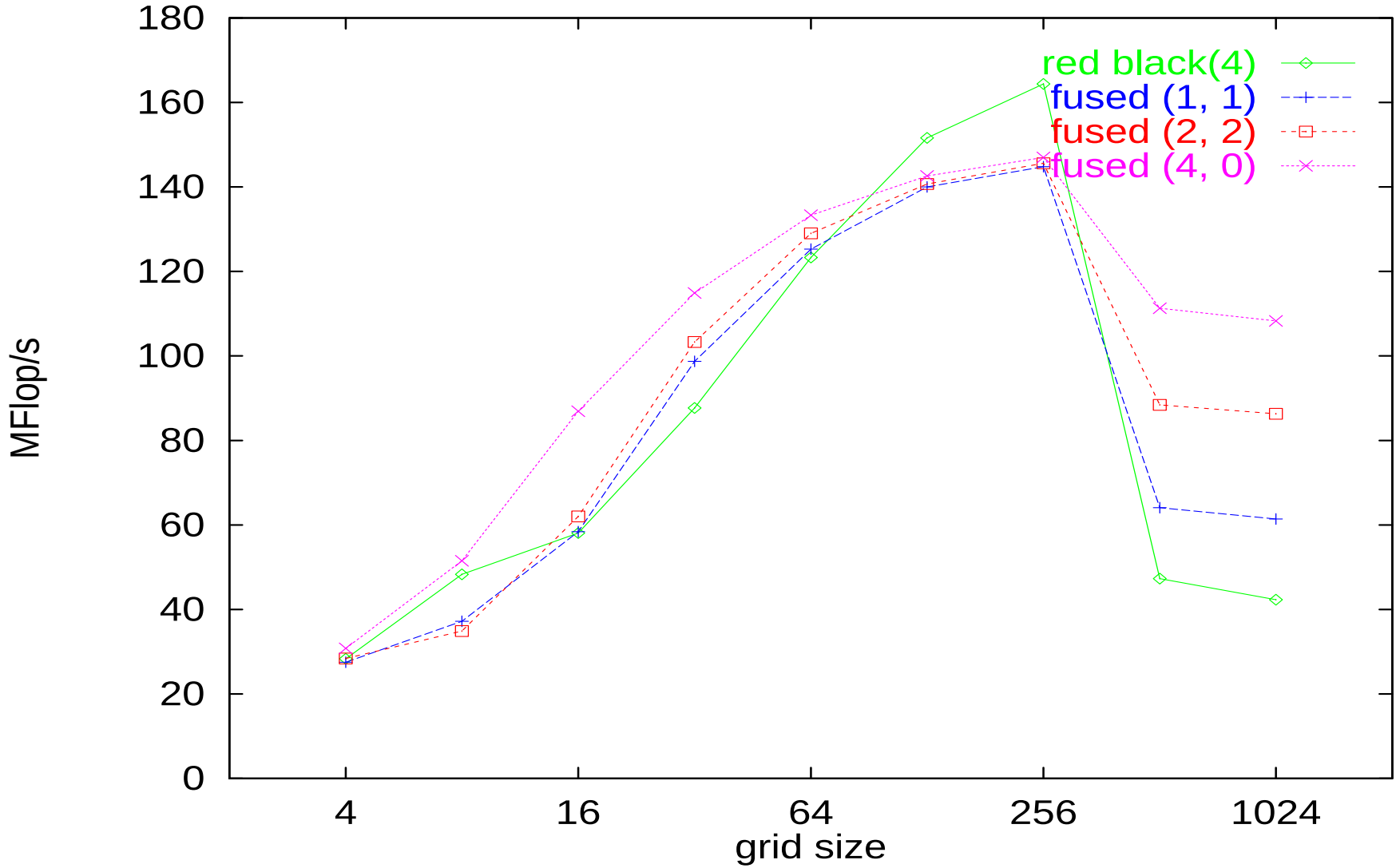
1. Update red points of line i
2. Update black points in line i-1
3. i=i+1 and goto step 1

Cache Aware Relaxation



1. Update black points of line i+2
2. Red points on lines i and i+1 could be updated
3. Update red and black points
4. Update red points of line i+4
5. Cascade down

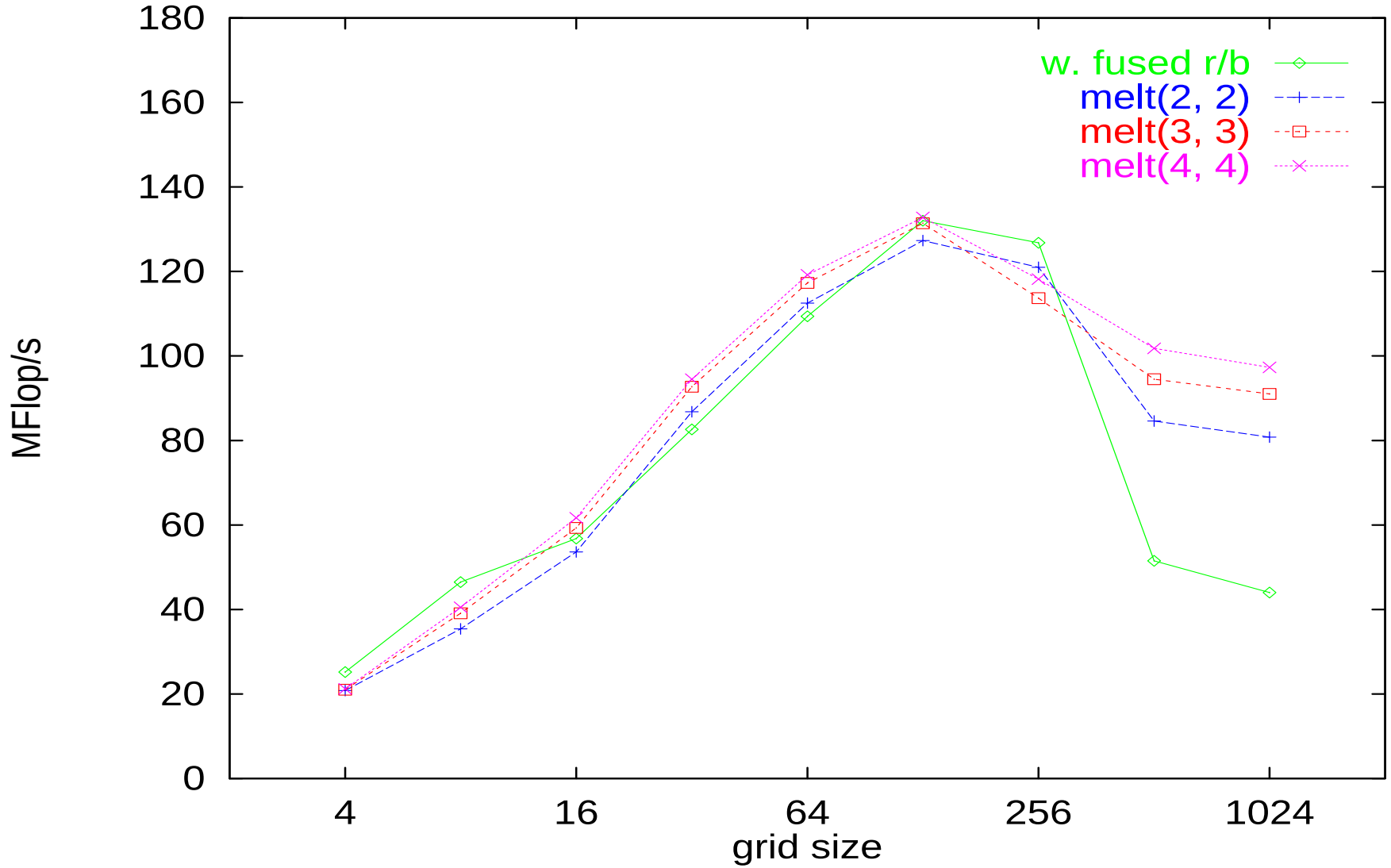
Fused R/B Results



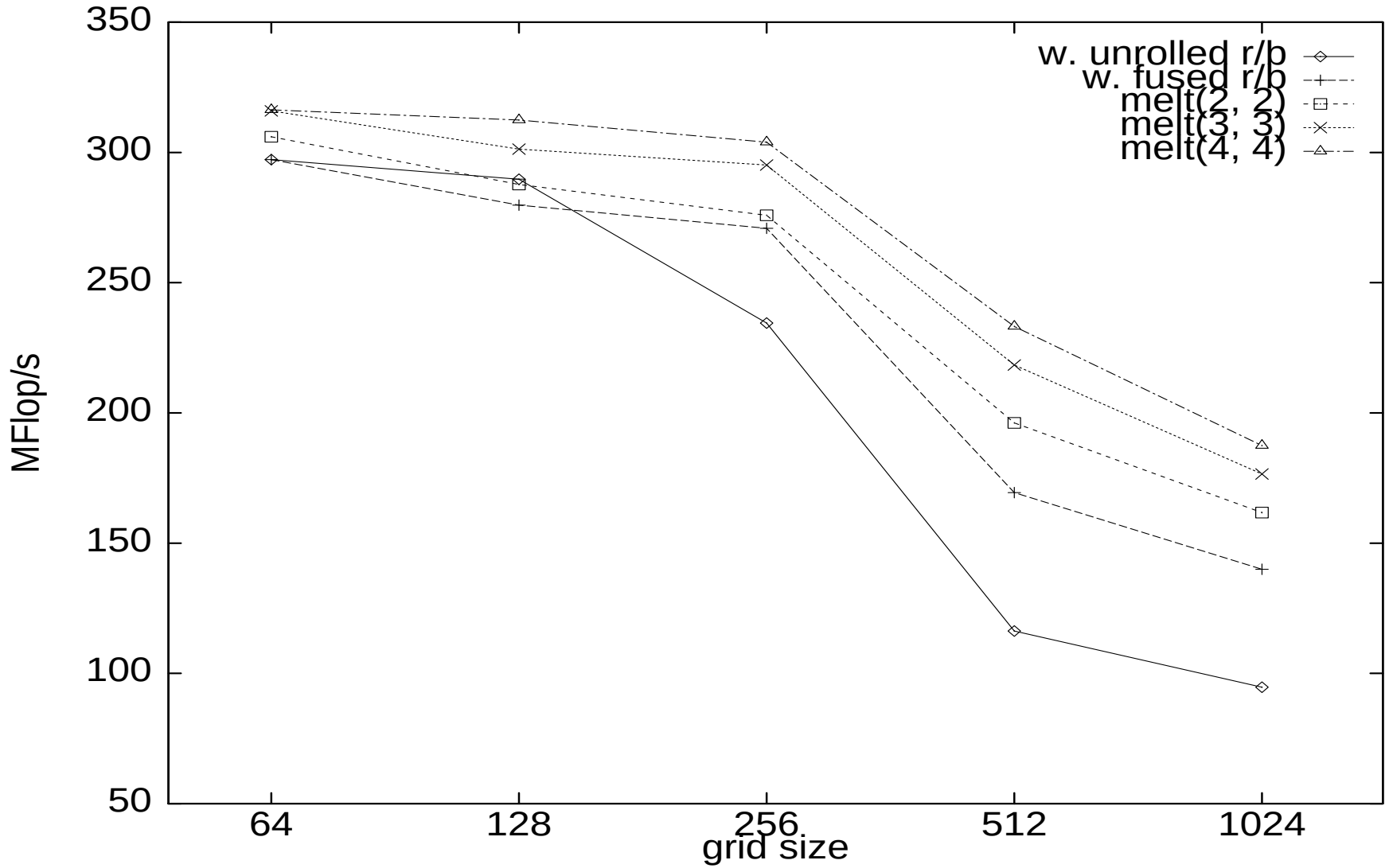
Cache Aware Multigrid

- melting pre coarse grid operations
  - pre smoother
  - residual calculation
  - restriction
- melting post coarse grid operations
  - interpolation
  - post smoother

Multigrid Results (SGI R8000)



Multigrid Results (DEC A21164)



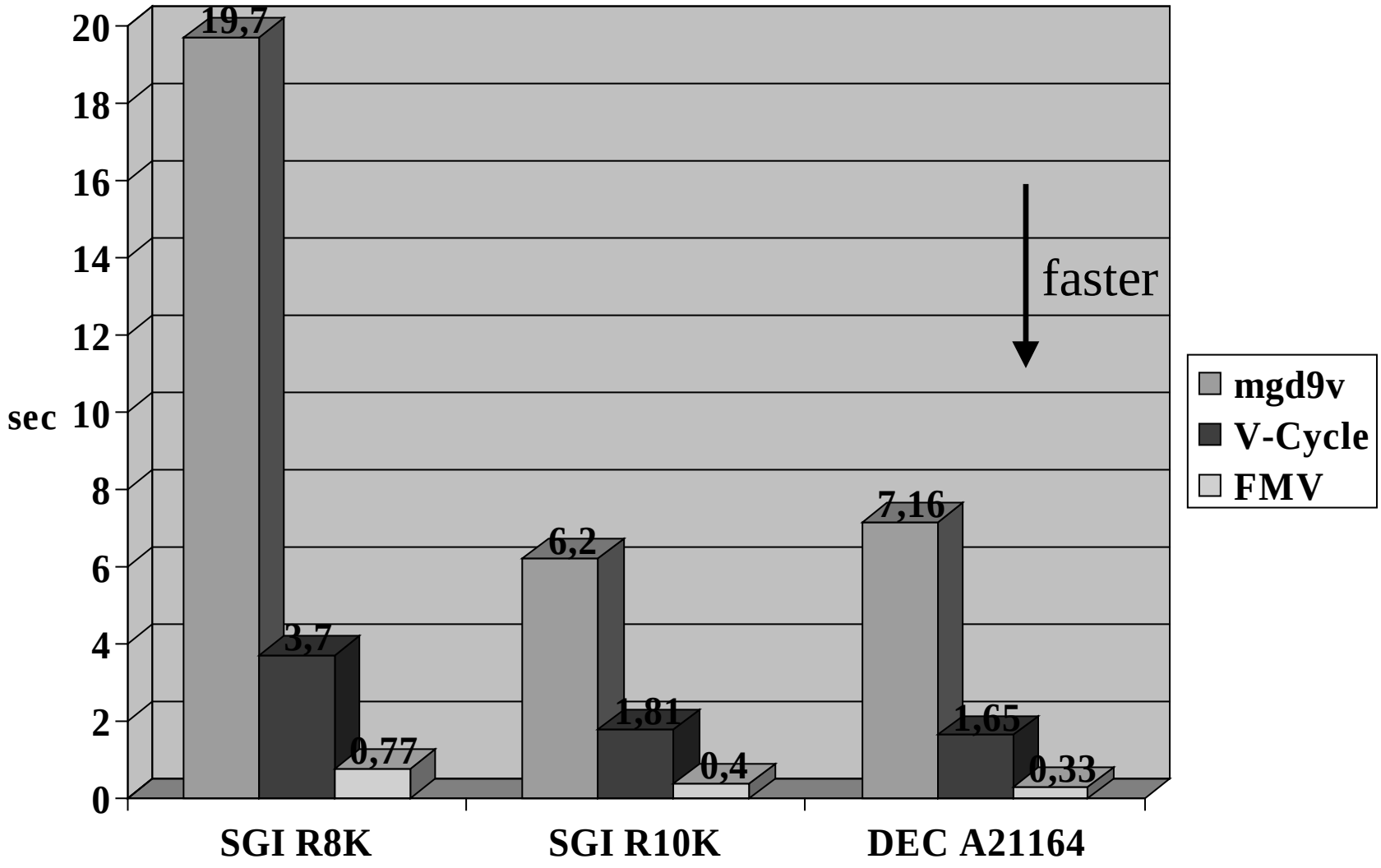
Where have all the Cycles gone ?

Performance Evaluation with DCPI on a DEC Alpha

- Evaluation of FMV Code on 1025x1025 grid
- 2 pre and post smoothers
- L_2-norm of residual < $4*10^{-8}$

	time	MFlop/s	% Cycles D-Miss	% Cycles Execution
Unrolled R/B	1.8 s	98	53.2	26.6
Fused R/B	1.3 s	142	31.7	37.1
Melt	1.1 s	162	28.6	38.5

Runtimes for 513 x 513 Grid Solution



Conclusions

- We optimized a straightforward multigrid code
  - transformation for in-cache optimizations
  - data locality transformation techniques
- 1025x1025 grid solutions
  - in ~1 second
  - exploiting up to 30% of peak performance
  - using C code
- data locality optimizations are still difficult

Further Work

- develop/use code restructuring tools
- remove the restriction of a constant stencil

  => allows integration into FEAST (S. Turek, Heidelberg)
- extend the ideas to 3D
- develop a tiling approach for the smoother